

ULTRASONIC IMAGING AND THE LONG WAVELENGTH PHASE

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INTRODUCTION

Elastodynamic and acoustic wave scattering play an essential role in various inspection methods such as sonar and ultrasonic tomography. Recently there has been considerable interest in the implications of long wavelength elastodynamic scattering for the characterization of flaws in elastic solids [1-6]. If the scattering amplitude is expanded as a power series in the frequency, the leading term is real and varies as the frequency squared. The next term varies as the frequency cubed and is purely imaginary. The evaluation of the phase variation in the long wavelength limit requires the ratio of these terms. Most effort to date has been invested in understanding the dependence of the coefficient of the frequency squared term on the size, shape, orientation and material properties of the scatterer. Richardson [3] and Kohn and Rice [4] have shown that, for an anisotropic elastic inclusion in an otherwise isotropic and homogeneous elastic space, the coefficient depends on at most 22 parameters. In addition, efficient numerical programs have been constructed to evaluate this coefficient for ellipsoidal inclusions. Other work has related it to the stress intensity factor for flaws which are crack-like [5].

On the other hand, much less work has been done on establishing the features of the coefficient of the frequency cubed term and hence the long wavelength variation of the phase, ϕ_1 . Jones [7] has established a formalism for computing the long wavelength phase if the solutions to certain problems in elastostatics are known. Almost always these static problems are unsolved. Richardson [8,9] has shown that ϕ_1 may be made to vanish by an appropriate choice of the origin of coordinates: (1) if the scatterer possesses a center of inversion symmetry; or (2) if the density and elastic properties of the scatterer are nearly those of the host. In principle, information on the long wavelength phase variation might be extracted from various numerical calculations using the T-matrix method [10] or the method of optimal truncation [11]. To the best of our knowledge, no such systematic study has yet been carried out.

In this paper we will show that the long wavelength phase is opposite and equal for two scattering experiments, the second of which reverses the directions of incidence, scattering and the polarization of the transmitter and receiver.

The work reported in this paper is motivated by the need to develop practical ultrasonic imaging methods for the nondestructive evaluation of defects in structural solids. Ultrasonic imaging and inverse scattering methods are often implemented as follows. One insonifies the scatterer (flaw) with a broadband signal from one transducer and then records the signal with a second transducer. After the signal is stored and processed to remove the system response function, the transducers are moved and the process is repeated. Alternatively, arrays of transducers may be used. Once the flaw has been scanned from a variety of directions, the processed signals are added together in a coherent fashion and an image of the flaw is obtained.

For the imaging process to be successful, it is necessary that the various signals be coherent. That is, they must have a common phase reference. For small interior flaws in structural materials the establishment of such a coherent data set remains a substantial problem. Straightforward approaches generally fail due to the presence of small unknown anisotropies ($\sim 1-5\%$) in the velocity of sound in a given structural part.

The phase coherence problem just noted may be modeled as follows. Suppose that the scattering amplitude is known to within an overall phase error, $\exp(ik\tau)$. Here k represents the wavevector while τ is an unknown number which represents the phase error. It is assumed that τ differs for each transducer placement. The problem is then to determine τ , remove the phase error and then recover the scattering amplitude. The flaw's shape can then be recovered using standard imaging algorithms. Richardson [8,9] has shown that the above problem can be solved in two cases: (1) the flaw's material properties are nearly those of the host; or (2) the flaw has a center of inversion symmetry. His method depends on finding an origin of coordinates such that the long wavelength linear phase term is zero for all directions of incidence and scattering. Richardson's proposed phase recovery method and closely related variants have been successfully applied to experimental data by several groups [12,13,14]. In this paper we extend the validity of Richardson's observation to several new cases.

However, the most important result in this paper is a generalization of Richardson's work to the case of an arbitrary isolated flaw. No assumptions are made concerning the flaw's size, shape or scattering strength. This generalization does not allow a complete solution of the phase recovery problem. However, it does allow one to establish the relative phase of two signals when the directions of incidence, scattering and the polarization of the transmitter and receiver are reversed. This partial phase recovery allows one to establish an upper bound to the size of a wide class of flaws including voids and inclusions. For the purposes of nondestructive evaluation such bounds are often all that is needed.

The structure of this paper is as follows. In the next section elastodynamic scattering theory is reviewed using the formalism of Gubernatis et al. [15]. Then the long wavelength limit is discussed and the basic result is obtained.

REVIEW OF ELASTODYNAMIC SCATTERING

The theory of ultrasonic scattering is reviewed for a scattering center in an otherwise uniform and isotropic elastic space. The formulation and notation follows closely the development of Reference 15. The scattering geometry is shown in Figure 1. The transmitter launches a planar time harmonic displacement field. It propagates in the direction denoted by the unit vector \hat{e}^0 and is plane polarized in the \hat{a} direction.

The incident field interacts with the scatterer in a complex way. The displacement is then measured (in the far-field of flaw) on the surface of the sphere S . The radius of the sphere is taken in the limit that it becomes arbitrarily large. Consequently, the data, which are measured by a detector of finite area, can be characterized as propagating in the direction of scattering \hat{e}^s . The detector is plane polarized in the direction \hat{b} . It is assumed that the scatterer is confined to a region of space, R_2 ; that is, the scatterer has compact support.

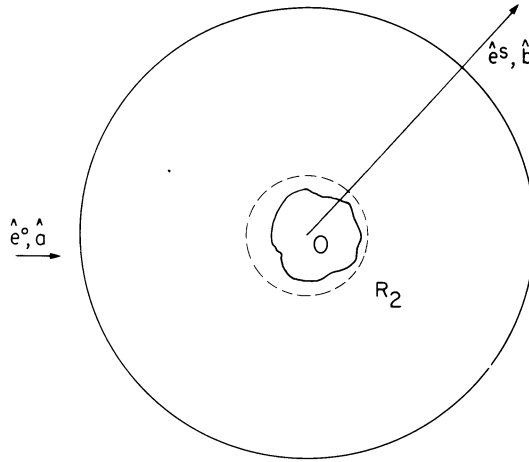


Figure 1. The scattering geometry. R_2 denotes the region of support for the material property deviations. The incident displacement field propagates in the direction \hat{e}^o with its polarization vector in the \hat{a} direction. The data are measured on the surface of the asymptotically large sphere S which is centered about the scattering region. The detector measures the displacement field propagating in the \hat{e}^s direction with polarization vector in the \hat{b} direction.

The material properties are denoted by

$$\rho(\vec{x}) = \rho^o + \delta\rho(\vec{x}) \quad (1a)$$

and

$$C_{ijkl}(\vec{x}) = C^o_{ijkl} + \delta C_{ijkl}(\vec{x}). \quad (1b)$$

Here ρ^o and C^o_{ijkl} are the constant density and isotropic elastic constants in the region exterior to R_2 . The host material is assumed to be isotropic and the Lamé parameters λ^o and μ^o determine C^o_{ijkl} . The quantities $\delta\rho(\vec{x})$ and $\delta C_{ijkl}(\vec{x})$ denote the deviations of the density and the elastic constants in the region of the scatterer; δC_{ijkl} need not be isotropic. It is assumed: (1) that $\delta\rho$ and δC_{ijkl} are such that ρ and C_{ijkl} are positive everywhere; and (2) ρ and C_{ijkl} are bounded everywhere. However, ρ and C_{ijkl} may have discontinuities.

For time harmonic waves, the propagation of waves of displacement is governed by

$$[C_{ijk\ell}(\vec{x})u_{k,\ell}(\omega,\vec{x})]_{,j} + \rho(\vec{x})\omega^2 u_i(\omega,\vec{x}) = 0; \quad (2)$$

and the boundary conditions. Here ω is the angular frequency and u_i is a vector denoting the displacement. Vector notation is used in the arguments when no risk of confusion exists. Otherwise, standard tensor notation is used and repeated indices indicate summation. An index preceded by a comma indicates the derivative with respect to the specified component of \vec{x} .

It is convenient to combine the equation of motion and the boundary conditions in the integral equation

$$\begin{aligned} u_i(\hat{e}^0, \hat{a}, \omega, \vec{x}) &= u_i^I(\hat{e}^0, \hat{a}, \omega, \vec{x}) \\ &+ \omega^2 \int d^3\vec{x}' g_{im}(\omega, \vec{x}-\vec{x}') u_m(\hat{e}^0, \hat{a}, \omega, \vec{x}') \delta\rho(\vec{x}') \\ &+ \int d^3\vec{x}' g_{ij,k}(\omega, \vec{x}-\vec{x}') u_{\ell,m}(\hat{e}^0, \hat{a}, \omega, \vec{x}') \delta C_{jk\ell m}(\vec{x}'). \end{aligned} \quad (3)$$

The primed subscript indicates the derivative with respect to the m^{th} component of \vec{x}' . The incident field is given by

$$u_i^I(\hat{e}^0, \hat{a}, \omega, \vec{x}) = \exp(i\gamma \hat{e}_i^0 x_i) \hat{a}_i \quad (4)$$

Here \hat{e}^0 and \hat{a} are unit vectors denoting, as previously remarked, the direction of propagation and polarization of the incident wave (which is assumed to be either completely longitudinally or completely transversely polarized). The magnitude of the wavevector is given by γ . For longitudinally (L) polarized incident waves, $\gamma \equiv \alpha = \omega/c_L$ and $\hat{a} = \hat{e}^0$. For transversely (T) polarized waves $\gamma \equiv \beta = \omega/c_T$ and $\hat{a} \perp \hat{e}^0$. Also, c_L and c_T denote the velocity of longitudinal and transverse displacement waves, respectively. It should be emphasized that the \vec{x}' variable in Eq. (3) spans only the scattering region, R_2 . Further, since (3) is an integral equation it can be solved for \vec{x} in R_2 .

The function g_{ij} appearing in Eq. (4) is the elastodynamic Green's function for the host material. It satisfies outgoing boundary conditions and is defined by

$$C^0_{ipk\ell} g_{kj,\ell p}(\omega, \vec{x}-\vec{x}') + \rho^0 \omega^2 g_{ij}(\omega, \vec{x}-\vec{x}') = -\delta_{ij} \delta(\vec{x}-\vec{x}'). \quad (5)$$

Explicitly

$$g_{ij}(\omega, \vec{x}-\vec{x}') = \frac{1}{4\pi\rho^0\omega^2} \left[\frac{\delta_{ij}\beta^2 e^{i\beta|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} - \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} \left[\frac{e^{i\alpha|\vec{x}-\vec{x}'|} - e^{i\beta|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} \right] \right] \quad (6)$$

The longitudinal and transverse scattering amplitude are defined by the asymptotic form of Eq. (3) as $|\vec{x}|$ becomes large

$$\begin{aligned} &[u_i(\hat{e}^0, \hat{a}, \omega, \vec{x}) - \hat{a}_i \exp(i\gamma \hat{e}_i^0 x_i)] \\ &= A_i(\hat{e}^0, \hat{e}^s, \hat{a}, \omega) \exp(i\alpha|\vec{x}|)/|\vec{x}| + \\ &+ B_i(\hat{e}^0, \hat{e}^s, \hat{a}, \omega) \exp(i\beta|\vec{x}|)/|\vec{x}| + O\left(\frac{1}{|\vec{x}|^2}\right) \end{aligned} \quad (7)$$

Here $\hat{e}^s = \vec{x}/|\vec{x}|$ is the direction of scattering. The scattering amplitude for longitudinally polarized waves is given by

$$A_i(\hat{e}^o, \hat{e}^s, \hat{a}, \omega) = \frac{\hat{e}_i^s}{4\pi\rho^o c_L^2} [\omega^2 \int d^3\vec{x}' \hat{e}_j^s u_j(\hat{e}^o, \hat{a}, \omega, \vec{x}') \delta\rho(\vec{x}') \exp(-i\alpha\hat{e}_n^s x_n') + \frac{i\omega}{c_L} \hat{e}_j^e \hat{e}_k^s \int d^3\vec{x}' u_{\ell, m'}(\hat{e}^o, \hat{a}, \omega, \vec{x}') \exp(-i\alpha\hat{e}_n^s x_n')]; \quad (8)$$

and the transverse scattering amplitude is given by

$$B_i(\hat{e}^o, \hat{e}^s, \hat{a}, \omega) = \frac{(\delta_{ij} - \hat{e}_i^s \hat{e}_j^s)}{4\pi\rho^o c_T^2} [\omega^2 \int d^3\vec{x}' u_j(\hat{e}^o, \hat{a}, \omega, \vec{x}') \delta\rho(\vec{x}') \exp(-i\beta\hat{e}_n^s x_n') + \frac{i\omega}{c_T} \hat{e}_k^s \int d^3\vec{x}' u_{\ell, m'}(\hat{e}^o, \hat{a}, \omega, \vec{x}') \delta C_{j k \ell m}(\vec{x}') \exp(-i\beta\hat{e}_n^s x_n')]. \quad (9)$$

These equations are the ones needed in the rest of the paper. They were taken from Ref. 15, which considers scatterers with constant material properties. However, the arguments in Ref. 15 can be straightforwardly extended and the equations given in this section are valid for scatterers with spatially varying material properties as well [16].

LONG WAVELENGTH PHASE

The strategy in this section is to: (1) expand the wave equation in powers of the frequency in the long wavelength limit; (2) show that the expanded displacement field transforms in a certain way under the transformation $(\hat{e}^o, \hat{a}) \rightarrow (-\hat{e}^o, -\hat{a})$; and (3) then show that the long wavelength scattering amplitude and phase behave as claimed.

The long wavelength scattering problem, in the formalism of Gubernatis et al, [1] requires the expansion of various quantities in powers of ω . Explicitly, for small ω

$$u_i(\hat{e}^o, \hat{a}, \omega, \vec{x}) = u_i^o(\hat{e}^o, \hat{a}, \vec{x}) + i\omega u_i^1(\hat{e}^o, \hat{a}, \vec{x}) + \omega^2 u_i^2(\hat{e}^o, \hat{a}, \vec{x}) + O(\omega^3), \quad (10a)$$

$$g_{ij}(\omega, \vec{x} - \vec{x}') = g_{ij}^o(\vec{x} - \vec{x}') + i\omega g_{ij}^1(\vec{x} - \vec{x}') + \omega^2 g_{ij}^2(\vec{x} - \vec{x}') + O(\omega^3), \quad (10b)$$

$$U_i^T(\hat{e}^o, \hat{a}, \omega, \vec{x}) = \hat{a}_i(1 + i\gamma \hat{e}_\ell x_\ell - \frac{1}{2}\gamma^2(\hat{e}_\ell x_\ell)^2) + O(\omega^3), \quad (10c)$$

$$A_i(\hat{e}^o, \hat{e}^s, \hat{a}, \omega) = A_i^2(\hat{e}^o, \hat{e}^s, \hat{a})\omega^2 + i\omega^3 A_i^3(\hat{e}^o, \hat{e}^s, \hat{a}) + O(\omega^4), \quad (10d)$$

and

$$B_i(\hat{e}^o, \hat{e}^s, \hat{a}, \omega) = B_i^2(\hat{e}^o, \hat{e}^s, \hat{a})\omega^2 + i\omega^3 B_i^3(\hat{e}^o, \hat{e}^s, \hat{a}) + O(\omega^4). \quad (10e)$$

The scattering amplitudes, A_i and B_i , behave asymptotically as $\omega^2(\omega \rightarrow 0)$ for flaws with compact support. This is characteristic of classical wave equations and Ref. 8 discusses this point for elastodynamic scattering.

Next, Eqs. (10a-10c) are substituted into the integral wave equation (Eq. 3) and terms are separated according to powers of ω . The result for the lowest order term is

$$u_i^o = \hat{a}_i + \int d^3\vec{x}' \delta C_{j k \ell m} g_{ij, k}^o u_{\ell, m'}^o \quad (12)$$

By iteration of Eq. (12) it is clear that $u_i^0 = \hat{a}_i$, and consequently, $u_{i,j}^0 = 0$. In the expansion of g_{ij}^1 one notes that g_{ij}^1 is a constant independent of $|\vec{x} - \vec{x}'|$ and thus $g_{ij,k}^1 = 0$. Using these results one obtains the first and second order terms in the expansion of the wave equation

$$u_i^1 = \frac{\gamma}{\omega} \hat{e}_n^0 x_n \hat{a}_i + \int d^3\vec{x}' \delta C_{jk\ell m} g_{ij,k}^0 u_{\ell,m}^1 \quad (13a)$$

and

$$u_i^2 = -\frac{1}{2} \frac{\gamma^2}{\omega^2} (\hat{e}_n^0 x_n)^2 \hat{a}_i + \int d^3\vec{x}' \delta \rho g_{im}^0 u_m^0 + \int d^3\vec{x}' \delta C_{jk\ell m} g_{ij,k}^0 u_{\ell,m}^2. \quad (13b)$$

Consider the transformation $(\hat{e}^0, \hat{a}) \rightarrow (-\hat{e}^0, -\hat{a})$; that is, reverse the direction and polarization of the incident field. One finds that u^0 , u^1 and u^2 behave as

$$u_i^0(\hat{e}^0, \hat{a}, \vec{x}) = -u_i^0(-\hat{e}^0, -\hat{a}, \vec{x}), \quad (14a)$$

$$u_i^1(\hat{e}^0, \hat{a}, \vec{x}) = u_i^1(-\hat{e}^0, -\hat{a}, \vec{x}), \quad (14b)$$

and

$$u_i^2(\hat{e}^0, \hat{a}, \vec{x}) = -u_i^2(-\hat{e}^0, -\hat{a}, \vec{x}). \quad (14c)$$

Higher order terms do not in general transform in this simple way.

The long wavelength formulae for the scattering amplitudes are obtained by expanding Eqs. (8-9) for A_i and B_i in powers of ω . The result for the longitudinally polarized scattering amplitude is

$$A_i^2 = \frac{\hat{e}_i^s}{4\pi\rho^0 c_L^2} \{ \hat{e}_j^s \hat{a}_j \int d^3\vec{x}' \delta \rho - \frac{1}{c_L} \hat{e}_j^s \hat{e}_k^s \int d^3\vec{x}' \delta C_{jk\ell m} u_{\ell,m}^1 \}. \quad (15a)$$

and

$$A_i^3 = \frac{\hat{e}_i^s}{4\pi\rho^0 c_L^2} \{ \hat{e}_j^s \int d^3\vec{x}' \delta \rho (u_j^1 - \frac{1}{c_L} \hat{a}_j \hat{e}_n^s x'_n) + \frac{1}{c_L} \hat{e}_j^s \hat{e}_k^s \int d^3\vec{x}' \delta C_{jk\ell m} (u_{\ell,m}^2 + \frac{1}{c_L} u_{\ell,m}^1 \hat{e}_n^s x'_n) \}. \quad (15b)$$

The transversely polarized scattering amplitudes are given by

$$B_i^2 = \frac{(\delta_{ij} - \hat{e}_i^s \hat{e}_j^s)}{4\pi\rho c_T^2} \left[\hat{a}_j \int d^3\vec{x}' \delta \rho - \frac{\hat{e}_k^s}{c_T} \int d^3\vec{x}' C_{jk\ell m} u_{\ell,m}^1 \right]. \quad (16a)$$

$$B_i^3 = \frac{(\delta_{ij} - \hat{e}_i^s \hat{e}_j^s)}{4\pi\rho c_T^2} \left[\int d^3\vec{x}' \delta \rho (u_j^1 - \frac{1}{c_T} \hat{e}_n^s x'_n \hat{a}_j) + \frac{\hat{e}_k^s}{c_T} \int d^3\vec{x}' \delta C_{jk\ell m} (u_{\ell,m}^2 + \frac{1}{c_T} u_{\ell,m}^1 \hat{e}_p^s x'_p) \right]. \quad (16b)$$

As previously noted, the receiving transducer measures the scattered displacement in the \hat{b} direction. The measured signals are thus proportional to $A = A_i \hat{b}_i$ and $B = B_i \hat{b}_i$. At long wavelength, we adopt the notation $A^2(\hat{e}^0, \hat{e}^s, \hat{a}, \hat{b}) = \hat{b}_i A_i^2(\hat{e}^0, \hat{e}^s, \hat{a})$, etc. Using Eqs. (14a-14c) in Eqs. (15a-b and 16a-b) one finds that

$$A^2(\hat{e}^o, \hat{e}^s, \hat{a}, \hat{b}) = A^2(-\hat{e}^o, -\hat{e}^s, -\hat{a}, -\hat{b}), \quad (17a)$$

$$A^3(\hat{e}^o, \hat{e}^s, \hat{a}, \hat{b}) = -A^3(-\hat{e}^o, -\hat{e}^s, -\hat{a}, -\hat{b}), \quad (17b)$$

$$B^2(\hat{e}^o, \hat{e}^s, \hat{a}, \hat{b}) = B^2(-\hat{e}^o, -\hat{e}^s, -\hat{a}, -\hat{b}), \quad (17c)$$

and

$$B^3(\hat{e}^o, \hat{e}^s, \hat{a}, \hat{b}) = -B^3(-\hat{e}^o, -\hat{e}^s, -\hat{a}, -\hat{b}), \quad (17d)$$

Equations (17a-d) are the fundamental results of this paper. The longitudinal and transverse phases are defined as

$$\phi_L = \tan^{-1} \left[\frac{\text{Im}(A)}{\text{Re}(A)} \right] \quad (18a)$$

and

$$\phi_T = \tan^{-1} \left[\frac{\text{Im}(B)}{\text{Re}(B)} \right] \quad (18b)$$

One finds that as $\omega \rightarrow 0$

$$\phi_L = \phi_L^o + \omega \phi_L^1 + O(\omega^2), \quad (19a)$$

$$\phi_T = \phi_T^o + \omega \phi_T^1 + O(\omega^2). \quad (19b)$$

here ϕ_L^o and ϕ_T^o are either zero or π depending on the relative signs of A^2 and A^3 and of B^2 and B^3 . The coefficients ϕ_L^1 and ϕ_T^1 are given by

$$\phi_L^1 = A^3/A^2 \quad (20a)$$

$$\phi_T^1 = B^3/B^2. \quad (20b)$$

Consequently,

$$\phi_L^1(\hat{e}^o, \hat{e}^s, \hat{a}, \hat{b}) = -\phi_L^1(-\hat{e}^o, -\hat{e}^s, -\hat{a}, -\hat{b}) \quad (21a)$$

$$\phi_T^1(\hat{e}^o, \hat{e}^s, \hat{a}, \hat{b}) = -\phi_T^1(-\hat{e}^o, -\hat{e}^s, -\hat{a}, -\hat{b}) \quad (21b)$$

Thus, the phase variation is opposite equal for two experiments, if the directions of incidence, scattering and the polarization of the transmitter and receiver are reversed in the second experiment.

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DISCUSSION

Mr. Birnbaum (NBS): There are all sorts of interesting variants in scattering problems. I mentioned to Jim the other day that we had looked at some that were suggested by the ideas used often in molecular physics. In molecular spectroscopy you use spectral moments as a variance, and can show that they are related to weighted averages over the volume of the sample. We have adapted the same approach to ultrasonic scattering and obtained similar results. However, let me add that we were able to do this only in the born approximation, which has its inaccuracies, but gave us some physical transparency because of its simplicity.

Mr. Rose: It would be very interesting to compare your work with that of John Richardson.

Mr. Birnbaum: For example, I believe the lowest moment gives us the volume --that's obvious--and then the odd moment, the next one, is zero, if you choose the center properly. I think that corresponds to your center of inversion symmetry--and then finally the next even moment gives you information about the shape, some averaged value that refers to the shape of the sample.